Alternative Approximation Concepts for Space Frame Synthesis

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A method for space frame synthesis based on the application of a full gamut of approximation concepts is presented. It is found that with the thoughtful selection of design space, objective function approximation, constraint approximation, and mathematical programming problem formulation options it is possible to obtain near-minimum mass designs for a significant class of space frame structural systems while requiring fewer than ten structural analyses. Example problems are presented which demonstrate the effectiveness of the method for frame structures subjected to multiple static loading conditions with limits on structural stiffness and strength.

Nomenclature

B	= number of design variables
E	= modulus of elasticity
$f(\bar{Z})$	= general function of $\bar{Z}$
$f_L(\bar{Z}), f_H(\bar{Z})$	= explicit linear and hybrid approximations, respectively, of $f(\bar{Z})$
$\bar{F}$	= element end forces
$g_q(\bar{Z}), \bar{G}(\bar{X},\bar{Y}), \bar{G}(\bar{Z})$	= behavior constraint functions
$\bar{H}(\bar{X},\bar{Y})$	= equality constraint functions
I, J	= number of RSPs and CSDs, respectively
L	= element length
$M(\bar{X},\bar{Y}), M(\bar{Z})$	= objective function (structural mass)
Q_R	= set of retained behavior constraints
N	= number of nodal displacements
$T(\bar{Z})$	= design element sizing variable recovery transformation
$\bar{u}$	= vector of nodal displacements
$\bar{X}, \bar{X}^U, \bar{X}^L; \bar{Y}, \bar{Y}^U, \bar{Y}^L$	= RSPs and CSDs, respectively; upper and lower bounds
$\bar{X}_0, \bar{Y}_0$	= RSPs and CSDs, respectively, at the beginning of a design stage
$\bar{X}(\bar{Z}), \bar{Y}(\bar{Z})$	= RSPs and CSDs, respectively, as implicit functions of the design variables
$\bar{Z}, \bar{Z}^U, \bar{Z}^L$	= generalized design variables, upper and lower bounds
$\bar{Z}_0$	= generalized design variables at the beginning of a design stage
$\{\Delta X_B\}, \{\Delta Y_B\}$	= changes in the dependent (basic) RSPs and CSDs, respectively, for a single design element
$\{\Delta X_F\}, \{\Delta Y_F\}$	= changes in the independent (free) RSPs and CSDs, respectively, for a single design element
ν	= Poisson's ratio
ρ	= mass density
σ_a, τ_a	= allowable normal and shear stress, respectively

Subscripts

b	= design variable
i, j	= RSP and CSD, respectively
k	= element end force
n	= nodal displacement
q	= retained constraint

Superscripts

$\bar{(\ )}$	= vector quantity
$\tilde{(\ )}$	= explicit approximation of the associated quantity
$(\ )^*$	= optimal quantity

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I. Introduction

DURING the past decade, optimization via general nonlinear mathematical programming techniques has become widely accepted as a viable methodology for engineering design. This has been particularly true in the structural engineering field.[1,2] Here, mathematical programming techniques have been coupled with finite-element-based structural analysis methods, through the application of various approximation concepts,[3,4] to yield a potentially powerful design tool.

While the basic methodology for structural synthesis is in place for a large class of problems, the majority of the reported computational experience has focused on truss- and membrane-type structures. There is, however, a significant class of problems for which a combined bending-membrane element representation must be used to adequately capture the essential structural behavior (e.g., frame-truss structures).

The extension of the synthesis methodology to the design of these structures has been slow and has met with only limited success. The principal difficulty encountered in this case has been that of choosing an appropriate set of design variables for which accurate behavior constraint approximations can be constructed while simultaneously maintaining adequate design freedom. This can be attributed to the fact that, while nodal displacements and rotations of truss-frame structures are well approximated as linear functions of section property reciprocals, combined stress and local buckling constraints for frame members generally depend upon element end forces, section properties, and cross-sectional dimensions in a nonlinear manner. Furthermore, for statically indeterminate structures, end forces for a particular element theoretically depend upon the section properties of all elements in the structure. Several approaches to this problem have been explored in previous work. The most popular of these approaches consists of selecting some or all of the element cross-section dimensions (CSDs), or their reciprocals, to be the structural design variables.[5-10] An alternative approach, which has received considerably less attention for design elements of general cross-sectional geometry, is to perform the structural design directly in terms of the element reciprocal section properties (RSPs).[11] Unfortunately, numerical experience with both methods reveals that convergence of the design process is, in many cases, quite slow and large numbers of structural analyses are required.

The objective of the work presented here is to develop a space frame synthesis methodology that will yield near-minimum mass designs while requiring that fewer than 10 structural analysis problems be solved. To this end, the approximation concepts approach is extended to include 1) CSD and RSP design space options, 2) linear and hybrid objective function and constraint approximations, 3) specialized selective constraint dependence options, 4) an approximate problem updating scheme, and 5) primal and dual mathematical programming problem formulations.

II. Problem Statement

The space frame synthesis problem may be stated as follows: seek a minimum mass design such that all pertinent measures of structural behavior and all structural design variables remain within specified limits. For the case where the structural topology, configuration, materials, and loading conditions are prescribed, the design problem can be stated mathematically as

$$\begin{gathered}\min_{\bar{X},\bar{Y}} M(\bar{X},\bar{Y})\\ \text{s.t.}\quad \bar{G}(\bar{X},\bar{Y}) \le \bar{0}, \qquad \bar{H}(\bar{X},\bar{Y}) = \bar{0}\\ \bar{X}^L \le \bar{X} \le \bar{X}^U, \qquad \bar{Y}^L \le \bar{Y} \le \bar{Y}^U \end{gathered} \tag{1}$$

where $\bar{X}$ and $\bar{Y}$ are vectors of element RSPs and CSDs, respectively, M is the structural mass and $\bar{G}$ a vector of behavior constraints. The equality constraints $\bar{H}(\bar{X},\bar{Y})$ have been introduced to account for any interdependence in the set of variables $\{\bar{X},\bar{Y}\}$. These constraints are highly nonlinear but can be separated into groups, each associated with a particular structural member. While Eq. (1) may be solved directly, an alternative approach is used here. It is possible to reduce the number of design variables in the problem and to eliminate the nonlinear equality constraints by rewriting Eq. (1) in terms of an independent set of generalized design variables $\bar{Z}$ as follows:

$$\begin{gathered}\min_{\bar{Z}} M(\bar{Z})\\ \text{s.t.}\quad \bar{G}(\bar{Z}) \le \bar{0}\\ \bar{X}^L \le \bar{X}(\bar{Z}) \le \bar{X}^U, \qquad \bar{Y}^L \le \bar{Y}(\bar{Z}) \le \bar{Y}^U \end{gathered} \tag{2}$$

where, in general, $\bar{Z}$ is some subset of $\{\bar{X},\bar{Y}\}$ determined as described in Sec. III.

In principle, the mathematical programming problem represented by Eq. (2) can be solved directly for $\bar{Z}$, with $\bar{X}$ and $\bar{Y}$ being subsequently determined via a recovery transformation of the form

$$\{\bar{X},\bar{Y}\} = T(\bar{Z}) \tag{3}$$

However, Eq. (2) represents a complex, implicit, nonlinear problem in terms of $\bar{Z}$ and as a result its direct solution is computationally impractical. A more tractable approach is to solve the design problem posed in Eq. (2) as a sequence of explicit approximate problems of reduced dimensionality. The construction and solution of each of these problems consist of the following phases: 1) approximate problem generation, 2) optimization, and 3) detail design recovery. Each of these steps is described in the following sections.

III. Approximate Problem Generation

Each approximate design problem is constructed through the application of a variety of techniques commonly referred to as approximation concepts.[3,4] These techniques form the basis for an approximate problem generation procedure which consists of the following steps: 1) constraint evaluation and deletion, 2) objective function and constraint approximation, and 3) selection of the independent generalized design variables.

Constraint Evaluation and Deletion

Two types of behavior constraints are considered here: 1) limits on overall structural stiffness in the form of nodal displacement and rotation constraints, and 2) limits on element strength in the form of stress and local buckling constraints. These constraints may be written in terms of the structural response quantities $(\bar{u},\bar{F})$ and the design element CSDs and RSPs as

$$g_q(\bar{Z}) = g_q(\bar{u}(\bar{X})) = R_q(\bar{u}(\bar{X})) - 1 \le 0, \qquad q \in Q_1 \tag{4}$$

for the displacement constraints, and

$$\begin{aligned} g_q(\bar{Z}) &= g_q(\bar{F}(\bar{u}(\bar{X})),\bar{X},\bar{Y})\\ &= R_q(\bar{F}(\bar{u}(\bar{X})),\bar{X},\bar{Y}) - 1 \le 0, \quad q \in Q_2 \end{aligned} \tag{5}$$

for the strength constraints, where R_q is the ratio of the qth behavior quantity to its associated allowable and R_q approaches unity as the behavior constraint becomes critical. The response quantities required for the evaluation of Eqs. (4) and (5) are computed using the well-known finite element displacement method. Detailed formulations of strength constraints (combined stress and local buckling) for several design element types (e.g., *I*-section, thin-walled circular tube, see Fig. 1) are given in Ref. 12, Appendix C.

Since the proper design of a structural system usually requires the consideration of a substantial number of failure modes, the structural synthesis problem statement frequently contains large numbers of inequality constraints. In order to reduce the number of constraints and the associated computational burden, it is possible to temporarily delete certain constraints that are not expected to currently participate in the design process. The criteria by which particular constraints are judged to be participating (active) or nonparticipating (passive) form the basis of the constraint deletion technique. A relatively simple, but effective, strategy consists of temporarily deleting all constraints with response ratios less than some specified constraint truncation parameter, CTP. Here,

$$\text{CTP} = \min\{\max\{R_c, 0.3\}, 0.7\} \tag{6}$$

where R_c is the maximum response ratio rounded down to the nearest tenth (e.g., if $\max\{R_q | q \in Q\} = 0.65$ then $R_c = 0.6$). Since R_c as well as all other response ratios R_q are updated at the beginning of each approximate problem generation phase, individual constraints may shift in and out of the retained set of participating constraints.

Objective Function and Constraint Approximations

A key element in the efficient solution of the structural synthesis problem is the construction of accurate explicit function approximations. This is particularly true in the case of the behavior constraint functions, since exact evaluation of these constraints requires that the structural analysis problem

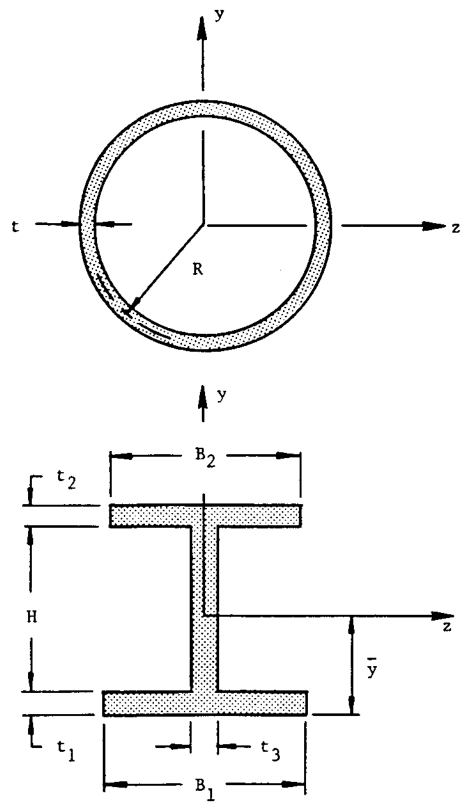

Fig. 1 Element cross-sectional types.

be solved. Various methods are available for the construction of these approximations, with those most commonly used requiring only first derivatives of the functions to be approximated.[4,13] Two types of first-order approximations are used here.

The first type of approximation consists of expanding the function in a linear first-order Taylor series of the form

$$f(\bar{Z}) \cong f_L(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial Z_b}(Z_b - Z_{0_b}) \quad (7)$$

where the expansion variables $\bar{Z}$ are chosen so that the resulting approximation is of the highest quality possible. In many cases, however, it is not possible to select $\bar{Z}$ such that all function approximations are of sufficient quality. In such cases, the following hybrid approximation[14] can be used.

$$f(\bar{Z}) \cong \tilde{f}_H(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial Z_b} B_b \quad (8)$$

where

$$B_b = (Z_b - Z_{0_b}) \qquad \text{if } \frac{\partial f(\bar{Z}_0)}{\partial Z_b} > 0$$

$$= -Z_{0_b}^2\left[(1/Z_b) - (1/Z_{0_b})\right] \qquad \text{if } \frac{\partial f(\bar{Z}_0)}{\partial Z_b} < 0 \quad (9)$$

This approximation is conservative as compared to alternative first-order approximations using either direct or reciprocal expansion variables and, in many cases, provides a better approximation to the function than Eq. (7).[15]

The construction of both the linear and hybrid approximations previously described requires the calculation of the first derivatives of the function to be approximated. In the case of the objective function, these derivatives are easily evaluated. However, the derivatives of the behavior constraints are considerably more complicated. For the stiffness constraints, differentiation of Eq. (4) with respect to the bth generalized design variable yields

$$\frac{\partial g_q}{\partial Z_b} = \sum_{i=1}^{I} \frac{\partial g_q}{\partial X_i}\frac{\partial X_i}{\partial Z_b} = \sum_{i \in I_b} \frac{\partial g_q}{\partial X_i}\frac{\partial X_i}{\partial Z_b} \quad (10)$$

where

$$\frac{\partial g_q}{\partial X_i} = \sum_{n=1}^{N} \frac{\partial g_q}{\partial u_n}\frac{\partial u_n}{\partial X_i} = \sum_{n \in N_q} \frac{\partial g_q}{\partial u_n}\frac{\partial u_n}{\partial X_i} \quad (11)$$

and where I_b is the set of RSPs associated with the bth generalized design variable and N_q is the set of displacement degrees of freedom associated with the qth stiffness constraint. In the case of displacement constraints involving a single degree of freedom, the summation over $n \in N_q$ reduces to a single term. For the strength constraints, differentiation of Eq. (5) with respect to Z_b gives

$$\frac{\partial g_q}{\partial Z_b} = \sum_{i \in I_b} \frac{\mathrm{d} g_q}{\mathrm{d} X_i}\frac{\partial X_i}{\partial Z_b} + \sum_{j \in J_b} \frac{\partial g_q}{\partial Y_j}\frac{\partial Y_j}{\partial Z_b} \quad (12)$$

where

$$\frac{\mathrm{d} g_q}{\mathrm{d} X_i} = \sum_{k \in K_q} \frac{\partial g_q}{\partial F_k}\frac{\partial F_k}{\partial X_i} + \frac{\partial g_q}{\partial X_i} \quad (13)$$

and where J_b is the set of CSDs associated with the bth generalized design variable and K_q is the set of design element end forces associated with the qth strength constraint.

For given forms of the constraint functions, and given the relationships between the design variables and the design element CSDs and RSPs, Eqs. (10–13) can be evaluated, yielding the behavior constraint derivatives. However, it is necessary to first obtain the structural response quantity sensitivities ($\partial u_n/\partial X_i$, $\partial F_k/\partial X_i$). Various methods are available for computing these derivatives.[16] In this work, the nodal displacement sensitivities are obtained using the partial inverse form of the pseudoload method.[4,17] The design element end-force sensitivities are obtained via implicit differentiation of the element level force-displacement relations.[12]

A significant amount of the computational effort associated with the generation of the approximate design problem is expended during the calculation of the structural response quantity sensitivities. For many structural synthesis problems, significant reductions in computational effort can be realized by using a selective constraint dependence technique.[6] This technique is based on the observation that in many practical design problems a single behavior constraint may be strongly dependent on only relatively few design elements.

The design element strength constraints are well suited to the application of three special cases of the selective constraint dependence concept. Examination of Eqs. (5), (12), and (13) reveals that strength constraints for a given design element are coupled to the design variables associated with other elements only through the element end-force sensitivities. Therefore, it is possible to apply the selective constraint dependence technique via assumptions made as to the expected nature of any element force redistribution that may occur during the design process. The following three assumptions are considered here: 1) the element forces are invariant during the current stage in the design process, 2) changes in the forces on a given design element are primarily dependent on the design variables associated with that element, and 3) the element forces are strongly dependent on all of the structural design variables. These assumptions lead to the following hierarchy of element force sensitivity calculations: 1) no element force derivatives are calculated, 2) element force derivatives are calculated only with respect to the RSPs associated with that element, and 3) element force derivatives are calculated with respect to the RSPs of all elements.

Design Variable Selection

The final step of the approximate problem generation procedure is the actual selection of the independent generalized design variables. Two design variable selection schemes are used here. In the first case, the design element cross-sectional dimensions are chosen as the design variables (CSD design space). Optionally, a linearly independent subset of $\{\bar{X}, \bar{Y}\}$ may be chosen, with priority given to the independent RSPs (RSP design space). In either case, the number of design variables chosen is equal to the number of independent design element cross-sectional dimensions.

In conjunction with the selection of the design variables, relationships between the design variables ($\bar{Z}$) and the design element CSDs ($\bar{Y}$) and RSPs ($\bar{X}$) must be constructed for use in the formulation of the behavior constraint approximations [Eqs. (7–13)] and the side constraints [Eq. (2)]. The actual relationship between the generalized design variables and the element CSDs/RSPs is complicated and highly nonlinear. The efficient solution of the approximate design problems is facilitated by replacing this relationship with a linear approximation. This approximation is constructed by relating the first-order changes in the CSDs and RSPs to the corresponding changes in the design variables as follows:

$$[\{\Delta X_B\}\{\Delta Y_B\}\{\Delta X_F\}\{\Delta Y_F\}]^T = \left[\frac{\partial X_B}{\partial Z}\,\frac{\partial Y_B}{\partial Z}\,\frac{\partial X_F}{\partial Z}\,\frac{\partial Y_F}{\partial Z}\right]^T \{\Delta Z\} = [[H][I]]^T\{\Delta Z\} \tag{14}$$

where the subscripts B and F denote basic (dependent) and free (independent) quantities, respectively. The transformation matrix $[H]$, which relates the dependent CSDs and RSPs to the design variables, is constructed from the first-order relationship between $\bar{X}$ and $\bar{Y}$.[12]

Updating the Approximate Problem

The extent to which it is possible to minimize the number of structural analyses and response quantity sensitivity calculations required during the design process depends on the quality of the approximate design problems. Approximate problem statements for which the underlying constraint approximations are of high quality are valid over larger changes in the design variables and, as a result, fewer such problems are required to obtain the solution to the actual synthesis problem. The generation of the response quantity sensitivities in terms of the element RSPs yields derivative values which are accurate for relatively large changes in the design variables. This tends to improve the quality of the behavior constraint approximations, particularly for displacement constraints. However, the net quality of the constraint approximations depends not only on the response quantity sensitivities but also on the approximated relationships between the element CSDs/RSPs and the generalized design variables [see Eq. (14)]. Frequently, these relationships are accurate only for small changes in $\bar{X}$ and $\bar{Y}$. Hence, the net approximations may be accurate over smaller than desired changes in the design variables. Significant computational savings may be realized by periodically updating the approximate problem through recalculation of the matrix $[H]$ in Eq. (14). This may be done without recourse to either structural reanalysis or response quantity sensitivity calculations using the following procedure.

1) Calculate the approximate behavior constraint values for the pth update using the equation

$$g_q^p = g_q^{p-1} + \sum_{i=1}^{I} \frac{\partial g_q(\bar{X}^{p-1})}{\partial X_i}\left(X_i^p - X_i^{p-1}\right) + \sum_{j=1}^{J} \frac{\partial g_q(\bar{Y}^{p-1})}{\partial Y_j}\left(Y_j^p - Y_j^{p-1}\right) \tag{15}$$

2) Calculate the new objective function derivatives $\partial M/\partial X_i, i \in I$.

3) Calculate the new $[H]$ matrix in Eq. (14).

4) Calculate the new objective function and constraint derivatives using the equations

$$\frac{\partial M}{\partial Z_b} = \sum_{i \in I_b} \frac{\partial M(\bar{Z}_0)}{\partial X_i}\frac{\partial X_i}{\partial Z_b}, \quad b = 1,2,\ldots,B$$

$$\frac{\partial g_q}{\partial Z_b} = \sum_{i \in I_b} \frac{\partial g_q(\bar{Z}_0)}{\partial X_i}\frac{\partial X_i}{\partial Z_b} + \sum_{j \in J_b} \frac{\partial g_q(\bar{Z}_0)}{\partial Y_j}\frac{\partial Y_j}{\partial Z_b} \tag{16}$$

$$q \in Q_R, \quad b = 1,2,\ldots,B \tag{17}$$

where the quantities $\partial M(\bar{Z}_0)/\partial X_i$, $\partial g_q(\bar{Z}_0)/\partial X_i$, and $\partial g_q(\bar{Z}_0)/\partial Y_j$ were computed and saved during the previous approximate problem generation.

5) Construct the new objective function and constraint approximations using Eqs. (7–9).

In principle, using the procedure outlined above, the approximate problem can be updated and solved repeatedly. However, in practice, the number of times this update procedure can be performed depends on the quality of the response quantity sensitivities. Therefore, it is of paramount importance that these sensitivities be generated directly in terms of those variables which will yield gradients of the highest quality possible, regardless of the final choice of design variables.

IV. Optimization

The approximate problem generation techniques discussed previously allow the implicit nonlinear frame synthesis prob-

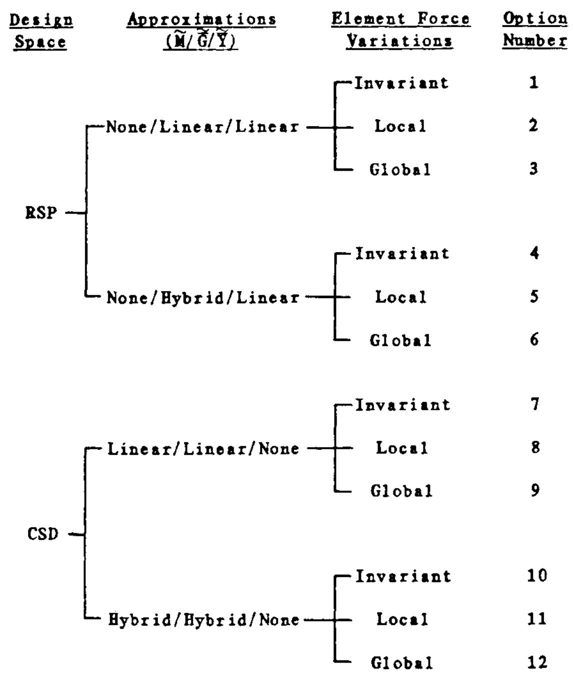

Fig. 2 Approximate problem formulation options.

lem posed in Eq. (2) to be replaced by a sequence of explicit approximate design problems, each having the form

$$\min_{\bar{Z}} \tilde{M}(\bar{Z})$$

$$\text{s.t.} \quad \tilde{g}_q(\bar{Z}) \leq 0, \; q \in Q_R; \qquad \bar{\bar{Z}}^L \leq \bar{Z} \leq \bar{\bar{Z}}^U \tag{18}$$

where $\tilde{M}$ and $\tilde{g}_q$ are explicit approximations of the structural mass and retained constraints, respectively, and where $\bar{\bar{Z}}^L$ and $\bar{\bar{Z}}^U$ are the upper and lower bounds on $\bar{Z}$. These bounds are constructed from move limits on $\bar{Z}$ to ensure the validity of the behavior constraint approximation. Twelve different approximate problem statements are available depending on the choice of design space as well as objective function, behavior constraint and side constraint approximations, see Fig. 2. Each of these problems represents an explicit, separable, convex inequality constrained mathematical programming problem, and, as such, may be solved via any number of well-known nonlinear minimization techniques.[18] Two solution methods are implemented here.

The first solution option is a feasible directions method[19] as implemented in the CONMIN[20] computer program. This primal method serves as the primary solution technique and can be used to solve all 12 approximate problem forms. The method is reliable and relatively efficient for the class of problems considered here.

An alternative method for solving the mathematical programming problem posed in Eq. (18) is to replace this primal problem by the dual mathematical programming statement and solve the resulting dual maximization problem.[21-23] In this work, dual function maximization is carried out using the CONMIN program. Since CONMIN is a gradient-based algorithm, solution of the dual function maximization problem can only be guaranteed (see Ref. 12, p. 21) for formulation options 10–12 of Fig. 2. Numerical experience indicates that the dual method is more efficient than the primal method when the number of retained constraints is less than 1.5 times the number of primal design variables.

V. Detail Design Recovery

Having constructed and solved the approximate design problem as described previously, the final phase of the synthesis procedure consists of recovering the actual detail design quantities (sizing variables) from the generalized design variables ($\bar{Z}$). When the relationships between the sizing and design variables are simple and explicit (as is usually the case in truss design problems), the detailed design recovery process is trivial and rarely mentioned as a distinct part of the sequence of approximations approach to structural optimization. However, when this relationship is not explicit, as in the case where the RSP design space option is used and the sizing variables are the element CSDs, then the detail design recovery process can be quite complex and must be treated as a separate phase of the structural synthesis methodology.

Under the assumption that the element sizing variables are the CSDs, the detail design recovery process can be viewed as a procedure for calculating $\bar{Y}$ from the vector of optimal generalized design variables ($\bar{Z}^*$). In general, such a procedure seeks the solution of the set of nonlinear equations

$$\bar{Z}(\bar{Y}) = \bar{Z}^* \tag{19}$$

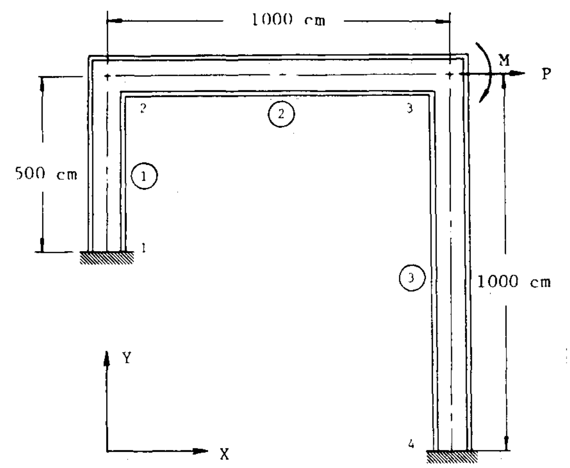

Fig. 3 Problem 1—portal frame.

Table 1 Design comparisons for portal frame

Member no.	Sizing variable	Initial value, cm	Lower bound, cm	Upper bound, cm	Final designs, cm Ref. 23	Final designs, cm Run 5 Option 12(D)
1	B_1	30.0	5.0	100.0	13.000	11.260
	t_1	1.0	0.1	5.0	0.450	0.410
	H	50.0	50.0	100.0	74.900	78.210
	t_3	1.0	0.1	5.0	0.497	0.523
	B_2	30.0	10.0	100.0	12.100	10.170
	t_2	1.0	0.1	5.0	0.487	0.456
2	B_1	30.0	5.0	100.0	11.400	11.690
	t_1	1.0	0.1	5.0	0.404	0.417
	H	50.0	50.0	100.0	89.900	99.470
	t_3	1.0	0.1	5.0	0.397	0.435
	B_2	30.0	10.0	100.0	10.700	10.940
	t_2	1.0	0.1	5.0	0.435	0.447
3	B_1	30.0	5.0	100.0	7.500	5.000[a]
	t_1	1.0	0.1	5.0	0.268	0.143
	H	50.0	25.0	100.0	61.900	25.000[a]
	t_3	1.0	0.1	5.0	0.250	0.100[a]
	B_2	30.0	10.0	100.0	10.000[a]	10.000[a]
	t_2	1.0	0.1	5.0	0.369	0.276
Volume, cm^3					90,592	84,058

[a]Sizing variable at lower bound.

Table 2 Iteration history data for portal frame

Analysis no.	Run 1 Option 1(P)	Run 2 Option 2(P)	Run 3 Option 3(P)	Run 4 Option 12(P)	Run 5 Option 12(D)
	Volume, cm^3 (maximum constraint violation, %)				
0	275,000 [0]	275,000 [0]	275,000 [0]	275,000 [0]	275,000 [0]
1	193,179 [0]	193,179 [0]	193,179 [0]	139,170 [85.6][a]	108,464 [> 100][a]
2	126,224 [84.0][a]	126,224 [84.0][a]	126,224 [84.0][a]	105,446 [53.4][a]	95,044 [31.1]
3	119,299 [17.6]	133,432 [10.9]	121,745 [21.5]	100,449 [0]	88,177 [41.2][a]
4	102,977 [> 100.0][a]	119,212 [12.5]	102,563 [> 100.0][a]	92,640 [> 100][a]	86,557 [0]
5	101,275 [55.1]	101,976 [> 100][a]	101,018 [46.1]	87,766 [16.8]	84,766 [0]
6	104,134 [1.0]	99,087 [70.3]	97,564 [8.6]	85,898 [0]	84,238 [0.8]
7	99,506 [> 100.0][a]	100,041 [12.9]	97,444 [0.3]	85,417 [0]	84,109 [0.7]
8	86,049 [> 100.0][a]	100,527 [0.8]	97,459 [0.2]	84,342 [0]	84,022 [0.5]
9	94,010 [> 100.0][a]	100,547 [0.1]	97,460 [0]	84,302 [0]	84,056 [0.7]
10	91,575 [> 100.0]	99,083 [1.7]		84,183 [0]	84,058 [0.3]
11	93,696 [> 100.0]	99,110 [0.1]		84,183 [0]	
12	93,575 [37.2]	97,880 [1.3]		84,157 [0]	
13	93,955 [0.2]	97,934 [0]			
14	94,091 [0.1]	97,560 [0.6]			
15	93,702 [0.2]	97,675 [0]			
16	93,773 [0.1]	97,574 [0]			
17	93,732 [0.3]	97,226 [0.6]			
18		97,365 [0]			
19		97,239 [0]			
20		97,240 [0]			

[a] Constraint was not retained during design stage.

subject to the following restrictions on the element CSDs

$$\bar{Y}^L \leq \bar{Y} \leq \bar{Y}^U \tag{20}$$

For the case where the RSP design space option is employed, the recovery process must attempt to solve Eq. (19) [subject to the constraints represented by Eq. (20)] directly. It should be recognized the Eq. (19) may not possess a solution within the acceptable domain defined by Eq. (20) and, therefore, any potential solution procedure must be able to deal with this possibility. As a result, the design recovery procedure for the RSP design space option will, in general, be approximate.

The recovery procedure used here makes direct use of the previously constructed approximate linear relationships between the changes in the element CSDs $\{\Delta Y\}$ and the changes in the generalized design variables $\{\Delta Z\}$ [see Eq. (14)]. This relationship can be written as

$$[\{\Delta Y_B\}\{\Delta Y_F\}]^T = \left[\left[\frac{\partial Y_B}{\partial Z}\right][I]\right]^T \{\Delta Z\} \tag{21}$$

The actual recovered values for the element CSDs are then given by

$$\{Y\} = \{Y_0\} + \{\Delta Y\} \tag{22}$$

where $\{Y_0\}$ contains the values of the element CSDs at the beginning of the design stage. Clearly, this procedure requires few additional computations and, therefore, can be applied efficiently to large problems. The main disadvantage of the method lies in the fact that Eq. (22) may be valid for only relatively small changes in the generalized design variables. However, this difficulty can be effectively overcome by using the approximate problem update technique described in Sec. III.

VI. Numerical Results

The frame synthesis methodology described previously has been implemented in a research computer program and installed on the IBM 3033 computer at the University of California-Los Angeles. Numerous example problems have been solved using this capability,[12] and some of these results are reported here.

Problem 1—Portal Frame

Figure 3 shows a three-member portal frame[24] subject to two independent loading conditions (L.C. 1: $P = 5.0 \times 10^4$ N, $M = 0$; L.C. 2: $P = 0$, $M = -2.0 \times 10^7$ N-cm). All of the members are made of the same material ($E = 7.0 \times 10^6$ N/cm^2, $G = 2.7 \times 10^6$ N/cm^2, $\nu = 0.3$, $\sigma_a = 2.0 \times 10^4$ N/cm^2, $\tau_a = 1.16 \times 10^4$ N/cm^2) and have the same type of symmetric I cross-sectional shape (see Fig. 1b). The structure is designed for minimum material volume subject to constraints on the lateral displacement and in-plane rotation at node 3, stress constraints at both ends of each member, local buckling constraints for the web and flanges of each member, and side constraints on the element CSDs. The displacement and rotation constraint allowables are given by $-4.0 \leq u_{3_x} \leq 4.0$ cm and $-0.015 \leq \theta_{3_z} \leq 0.015$ rad. The stress and local buckling constraint formulations are given in Refs. 12 and 24.

This design problem was solved via five different solution options using the initial design shown in Table 1. The iteration history data for these runs is given in Table 2. The iteration history plot for run 5 is shown in Fig. 4. In the first three runs [options 1(P), 2(P), and 3(P)], the design is carried out in the RSP design space (note that since the cross section has six CSDs the actual design variables include 4 RSPs and 2 CSDs) using linear approximations of the constraints. Each approximate problem is solved via a primal (P) solution method with move limits of 40% on the CSDs and 60% on the RSPs. Comparing the iteration histories for runs 1-3, it can clearly be seen that the superior results (in terms of the number of analyses required for convergence) are obtained in run 3 where the element end forces are assumed to be dependent on all of the structural design variables. This is not unexpected since the structure clearly has two competing load paths. While run 3 exhibits the best convergence, it does result in a final design that has a material volume 4% greater than obtained in run 1. Again, this is not too surprising, because this problem is known to possess several local minimum solutions.[24]

Based on the results of runs 1–3, runs 4 and 5 [options 12(P) and 12(D)] were made using the global element end-force variation option. The CSD design space option and hybrid

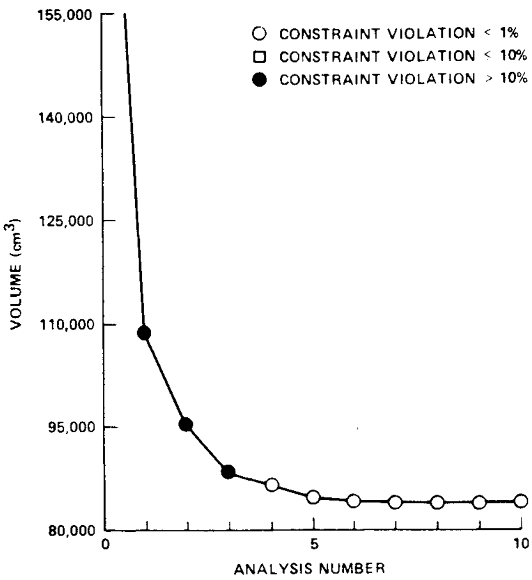

Fig. 4 Iteration history—problem 1, run 5.

behavior constraint approximations are employed. In both cases, 40% move limits are placed on the element CSDs. Each approximate problem is solved via a primal (P) solution method in run 4 and a dual (D) solution method in run 5. Comparison of the iteration histories for these runs reveals relatively little difference in convergence rate and overall maximum constraint violation for the intermediate designs. The final design material volume for run 5 is slightly less than that of run 4 but the design is also slightly infeasible. Comparison of the results of run 4-5 with runs 1-3 (see Ref. 12, Table 23) reveals a significant difference in the final design material volumes between those obtained from the CSD design space option and the RSP space (10.2–13.6%). While this is not unexpected, it is interesting to note that a distinct alternative design was obtained as a result of selecting a different design space.

The final design for run 5 is given along with that of the reference solution[24] in Table 1. Comparison of these designs, both in terms of final material volume and material distribution, clearly reveals the existence of at least two local minimum solutions associated with distinct design concepts. In the case of the reference solution, both primary load paths (members 1-2, member 3) contribute significantly to the load-carrying capacity of the structure. On the other hand, in run 5 the load path through member 3 is clearly abandoned in favor of the apparently more efficient path through members 1 and 2. It is interesting to note, however, that the same set of constraints is critical for both designs. These constraints include 1) the lateral displacement at node 3 and local buckling in member 1 for load condition 1, and 2) the in-plane rotation at node 3 and local buckling in members 2 and 3 for load condition 2.

Problem 2—Helicopter Tail Boom

Figure 5 depicts a space frame idealization of a helicopter tail boom[7] subject to a single loading condition. All members are made of the same material ($E = 10.5 \times 10^6$ psi, $G = 4.04 \times 10^6$ psi, $\nu = 0.3$, $\rho = 0.1$ lb/in.^{3}, $\sigma_a = 4.2 \times 10^4$ psi) and have the same tubular-type cross-sectional shape (see Fig. 1a). The structure is designed for minimum weight subject to constraints on nodal displacements in the y and z directions at nodes 5-28 and side constraints on the element CSDs. The displacement constraint allowables are given by $-0.5 \leq u \leq 0.5$ in. The side constraints are $0.25 \leq R \leq 25.0$ in. and $0.001 \leq t \leq 5.0$ in. for all members. Stress constraints are imposed at both ends of each member along with column buckling and

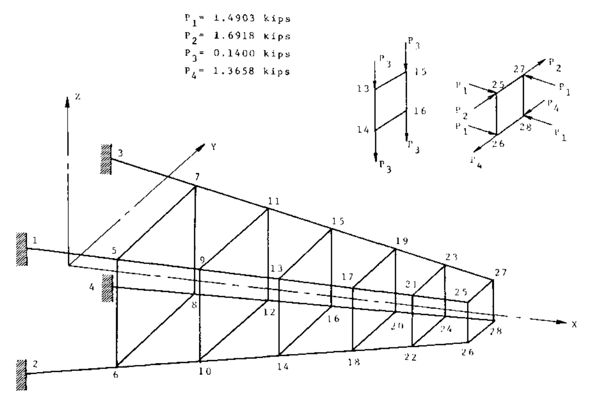

Fig. 5 Problem 2—helicopter tail boom.

Table 3 Element connectivity for helicopter tail boom

Member no.	Connectivity Node 1	Connectivity Node 2	Member no.	Connectivity Node 1	Connectivity Node 2
1	1	5	25	13	17
2	2	6	26	14	18
3	3	7	27	15	19
4	4	8	28	16	20
5	6	5	29	18	17
6	8	7	30	20	19
7	5	7	31	17	19
8	6	8	32	18	20
9	5	9	33	17	21
10	6	10	34	18	22
11	7	11	35	19	23
12	8	12	36	20	24
13	10	9	37	22	21
14	12	11	38	24	23
15	9	11	39	21	23
16	10	12	40	22	24
17	9	13	41	21	25
18	10	14	42	22	26
19	11	15	43	23	27
20	12	16	44	24	28
21	14	13	45	26	25
22	16	15	46	28	27
23	13	15	47	25	27
24	14	16	48	26	28

local wall buckling constraints (in the form of R/t constraints). The stress and buckling constraint formulations are given in Refs. 7 and 12.

This design problem was solved using five different solution options. For all cases, all members have an initial radius of 2.0 in. and wall thickness of 0.051 in. (see Table 3 for member connectivities). Since this problem is expected to be essentially displacement and R/t constrained,[7] all of the solution options chosen make use of the element end-force invariance assumption. The iteration history data is given in Table 4. The iteration history plots for run 1 (dashed line) and run 5 are shown in Fig. 6.

In the first two runs [options 1(P) and 4(P)] the design is carried out in the RSP design space using linear and hybrid behavior constraint approximations, respectively. Each approximate problem is solved via a primal solution method with move limits on the RSPs of 60% for run 1 and 70% for run 2. Comparison of the iteration histories for these runs reveals no difference in the convergence rate and only a slight change in the maximum constraint violation for the intermediate designs. It is interesting, however, to observe that, although the initial design is highly infeasible (211.6%), near-feasible designs are achieved after only three design stages.

In runs 3 and 4 [options 10(P) and 10(D)], the design is performed using the CSD design space option and hybrid

Table 4 Iteration history data for helicopter tail boom

	Weight, lb (maximum constraint violation, %)				
Analysis no.	Run 1 Option 1(P)	Run 2 Option 4(P)	Run 3 Option 10(P)	Run 4 Option 10(D)	Run 5 Option 1(PU)
0	69.11 [211.6]	69.11 [211.6]	69.11 [211.6]	69.11 [211.6]	69.11 [211.6]
1	95.86 [34.7]	102.73 [26.1]	99.10 [18.8]	97.87 [20.3]	105.44 [7.3]
2	107.94 [7.2]	107.32 [7.8]	109.46 [1.3]	108.65 [1.7]	112.34 [0]
3	110.69 [0.4]	109.44 [1.0]	110.00 [1.2]	109.62 [0]	110.20 [0]
4	109.51 [0.1]	109.51 [0.1]	110.04 [0.5]	108.74 [0.4]	109.15 [0]
5	110.13 [0]	109.97 [0]	109.44 [0.6]	108.34 [0.5]	108.90 [0]
6	109.33 [0]	109.55 [0]	108.96 [0]	108.74 [0]	108.79 [0]
7	109.25 [0]	109.60 [0]	108.79 [0]	108.66 [0]	
8	108.83 [0]	108.89 [0]	108.77 [0]	108.48 [0]	
9	108.71 [0]	108.89 [0]	108.73 [0]	108.52 [0]	
10	108.80 [0]	108.82 [0]		108.35 [0]	

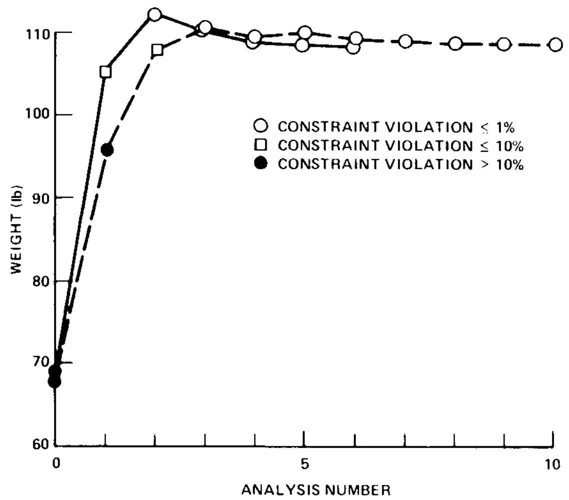

Fig. 6 Iteration history—problem 2, runs 1 and 5.

behavior constraint approximations. The approximate problems are solved via a primal method in run 3 and a dual solution method in run 4 with 40% move limits on the CSDs. There is little difference in convergence rate or maximum constraint violation for these runs, however, the dual solution method is less efficient here due to the large number of retained constraints.

The final run [option 1(PU)] for this problem is the same as run 1 except that the approximate problem update (U) procedure is employed. Here, the approximate problem is updated, without recourse to structural analysis or response quantity sensitivity calculations, once between each complete approximate problem generation. Comparison of the iteration history for this run with those of runs 1-4 reveals a dramatic improvement in both the convergence rate and the maximum constraint violation history.

The final designs for runs 1 and 5, along with those of the reference solution,[7] are given in Table 5. In all cases, the critical constraint set at the final design includes the displacement constraints at nodes 25 and 27 and the local wall buckling (R/t) constraints for nearly all of the members. Also, the final design weight and material distribution are nearly the same for all runs. The remarkable similarity in the material distribution is essentially due to the large number of critical R/t constraints, which effectively reduce the design freedom to one variable per member. Finally, it is interesting to observe the intuitively satisfying result whereby the lighter designs contain slightly larger members at the base (fixed) end of the structure.

Table 5 Design comparison for helicopter tail boom

			Final design, in.		
Linking group	Member nos.	Size var.	Ref. 7	Run 1 Option 1(P)	Run 5 Option 1(PU)
1	1-4	R	2.6695	3.1432	3.1198
		t	0.0880	0.0801	0.0794
2	5-8	R	1.9152	1.2850	1.3364
		t	0.0487	0.0355	0.0386
3	9-12	R	2.6530	2.8813	2.9110
		t	0.0829	0.0744	0.0737
4	13-16	R	2.1035	2.0071	1.9975
		t	0.0535	0.0511	0.0509
5	17-20	R	2.6784	2.8101	2.8239
		t	0.0753	0.0717	0.0717
6	21-24	R	2.1488	2.0703	2.0756
		t	0.0547	0.0527	0.0529
7	25-28	R	2.6238	2.6724	2.6672
		t	0.0673	0.0680	0.0678
8	29-32	R	2.1569	2.0965	2.0983
		t	0.0549	0.0535	0.0534
9	33-36	R	2.5038	2.5179	2.5103
		t	0.0637	0.0642	0.0640
10	37-40	R	2.1730	2.1475	2.1480
		t	0.0553	0.0548	0.0547
11	41-44	R	2.3748	2.3703	2.3618
		t	0.0604	0.0605	0.0602
12	45-58	R	1.9707	1.9487	1.9485
		t	0.0502	0.0497	0.0496
Weight, lb			111.20	108.80	108.70

VII. Conclusions and Recommendations

A synthesis methodology for the design of three-dimensional space frames subjected to multiple static loading conditions has been presented. This methodology has been implemented in a research program and used to solve a variety of frame design problems. The numerical results presented illustrate the feasibility of obtaining near-minimum mass designs after only 5-10 structural analysis problems have been solved.

The primary goal of this study has been achieved. However, it is important to realize that attaining this goal has required not only the introduction of a full gamut of approximation concepts, but also the proper application of these techniques to the problem at hand. Unlike the truss synthesis methodology, which tends to employ a rather standard set of approximation concepts to the solution of most problems with uniform success, the key to the efficient solution of a frame design problem, in many cases, lies in the thoughtful selection of appropriate approximation techniques and mathematical programming methods from a set of available options. A substantial body of computational experience for a set of test problems drawn from the literature is reported in Ref. 12 and, based on these results, guidelines for the selection of appropriate options for various types of problems are offered there.

Finally, while the frame synthesis methodology presented here can be used to solve a significant class of structural design problems, the following potential improvements can be identified: 1) expansion of the design element library to include additional cross-sectional shapes, 2) modification of the approximate problem update procedure to include updating of all explicit partial derivatives, and 3) replacement of the approximate linearized detail design recovery method with an efficient nonlinear technique.

Acknowledgment

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